Robert Hooke is commonly thought of as the inventor of ‘Hooke’s joint’ or the ‘universal joint’. However, it is shown that this flexible coupling (based on a four-armed cross pivoted between semicircular yokes attached to two shafts) was in fact known long before Hooke’s time but was always assumed to give an output exactly matching that of the input shaft. Hooke carefully measured the relative displacements of the two axes, and found that if one were inclined to the other, uniform rotation of the input produced a varying rate of rotation of the output. He also recognized that this variable rate exactly corresponded to the movement of the shadow of a gnomon across the face of a sundial, as generated by the projection of the uniform motion of the Sun around an inclined polar axis. He therefore proposed that a ‘mechanical sundial’ might be made by coupling a 24-hour clock movement (with its hour shaft at the appropriate inclination) to a pointer via a universal joint. This proposal has been investigated both practically and mathematically, and shown to be valid. Hooke’s studies of the universal joint caused it to be identified with his name, and it has ultimately proved far more important as a rotary coupling than as a sundial analogue. More complex versions subsequently designed by Hooke included provision for two basic couplings to be linked by an intermediate shaft. With appropriate setting of phase and shaft angles this ‘double Hooke’s joint’ could annul the variable output velocity characteristic of the single universal. It has proved invaluable for modern automotive transmissions.

**Keywords:** Robert Hooke; Hooke’s joint; universal; sundials; sundial-clock

**INTRODUCTION**

Robert Hooke (1635–1703) was appointed the first Curator of Experiments for the Royal Society in 1662, and was recognized for his many talents. However, the animosity of Isaac Newton, plus his own difficult personality and absence of family, led to his being rather neglected in his later years and overlooked after his death. This omission has been rectified in the past decade, with several biographies, books and conferences helping to restore Hooke’s reputation as a pioneering scientist, inventive engineer, gifted artist, and hard-working surveyor.

*am41@le.ac.uk*
Hooke and mechanisms

The quantitative study of technical mechanisms, and ideas for their application and improvement in novel scientific instruments, occupied a significant part of Hooke’s early professional life. Horology, astronomy and microscopy were particular favourites, but his name is now most commonly associated with the classic description of elasticity known as ‘Hooke’s law’. The second item to which his name is familiarly attached is ‘Hooke’s joint’, two of which are used in the transmission of almost every automobile.

Hooke and sundials

Sundials seem to have fascinated Robert from boyhood, because Aubrey records that while still at Freshwater ‘he made a Dial on a round trencher, never having had any instruction’. ‘Dialling’ was a subject that attracted many scholars in the sixteenth and seventeenth centuries, so several books would have been available to help him. The principle of the equal-hour sundial is that the Sun shining upon the edge (‘style’) of a triangular gnomon sloping upwards towards the celestial pole throws a shadow whose direction is independent of the varying declination of the Sun. It is controlled only by the hour angle, so a dial at right angles to the polar axis (an equatorial dial) may be divided into 24 equal 15° divisions to indicate the hours. The noon shadow falls in a vertical plane, coincident with the lowermost line on the dial. The situation is diagrammed in figure 1.

Figure 1. Construction of the hour lines on a sundial. (After Cousins.)

A. Mills
The shadow of the style may also be received upon other planes, horizontal or vertical being usual. However, it is clear from figure 1 that the angle with the polar axis (equal to the latitude with the horizontal dial and its complement with the vertical dial) results in the circular equatorial dial’s being projected as an ellipse. Consequently, the equality of the hour divisions characterizing that dial is lost, the hour lines now bunching symmetrically together more closely around noon (figure 2). The texts available in Hooke’s time prescribe various graphical methods for constructing dials to suit any latitude, as well as for planes at an angle to the horizontal (‘reclining’) or displaced from an exactly southern aspect (‘declining’).

Hooke’s interest in sundials is first on record when in November 1663 he suggested at a weekly meeting of the Royal Society that a perspective drawing device invented by Prince Rupert might be modified to describe all types of plane dials. Some years later, on 21 March 1667, he demonstrated a version made in wood to the assembled Fellows, depositing a drawing in the Register of the Society. It is reproduced here as figure 3, while figure 4 is a modern working reconstruction. It consists of two shafts attached at the midpoints of rectangular forks, the short sides of the forks being pivoted to opposite arms of a cross-shaped member held between them. Hooke claimed that if the ‘input’ shaft were supported at an angle equal to the latitude of a given site with respect to a horizontal ‘output’ shaft, then a pointer on the latter would indicate the hour lines of a vertical dial for that site if the input pointer were moved in equal 15° divisions. Hooke’s demonstration presumably satisfied the assembled Fellows, for the modern reconstruction agreed with dials constructed by recognized graphical or mathematical techniques. However, in the absence of any diagrams, Hooke’s explanation of the principle of his ‘sundial delineator’ is none too clear.

A MODERN ANALOGUE

A mechanical model devised by Chris Lusby-Taylor is shown in figure 5, and more clearly illustrates the way in which motion over the equatorial dial is resolved into motion over a plane inclined to it. A thin shaft is supported at a chosen angle (here 52°) to the horizontal base by a triangle of medium-density fibreboard and secured by screw-eyes that allow it to be
Figure 3. Robert Hooke’s first sundial delineator of 1667. (Copyright © The Royal Society.)

Figure 4. Modern working reconstruction of Hooke’s first sundial delineator.
rotated. This shaft simulates the polar axis, and a pointer inserted in a small hole drilled at right angles moves over a ‘clock face’ divided into 24 equal hours, with 12 noon at the top. A small spotlamp is also hinged to the inclined shaft, in line with the pointer. When illuminated, the shadow cast by the inclined shaft (also simulating the style of a gnomon) falls on a horizontal sheet of paper. The direction of the shadow is therefore independent of the angle of the lamp with respect to the shaft, but turning the latter as a whole causes the shadow to progress in a nonlinear manner around the paper ‘sundial’.

A thin wooden pointer of rectangular section is freely pivoted within a slot cut in the lower end of the sloping shaft. A conical depression at the centre of the paper dial both supports the pointed shaft and allows the lower pointer to slide over its surface. If the two pointers are set parallel it will be found that the second pointer accurately follows the shadow over the face of the dial, automatically resolving the motion of the ‘Sun’ over the ‘clock’ (or equatorial) dial into the corresponding motion in the horizontal plane. It will be realized that a similar mechanism could mimic the motion of the shadow of a gnomon over the lower half of a vertical sundial. One of the forks in Hooke’s delineator is represented by the slotted end of the polar shaft, and the pivoted pointer sliding over the paper dial takes the place of the other fork and its shaft.

**HooKE’S SUNDIAL DELINEATOR OF 1675**

In 1675, eight years after the announcement of his first sundial delineator, Robert Hooke took the opportunity offered by some unused space in the plates accompanying the publication of his third Cutlerian Lecture to illustrate\(^\text{18}\) an improved ‘instrument for describing all manner
of dials’ (figure 6). It incorporated the same cross-shaped member, but this was now pivoted between stirrup-shaped forks, allowing a greater range of inclination between the shafts. All components were carefully made from metal. As with its wooden predecessor, uniform rotation of the input shaft produced a nodding movement of the cross that gave rise to a variable output speed, decelerating from $0^\circ$ to $90^\circ$ and then accelerating from $90^\circ$ to $180^\circ$ before the cycle was repeated. Hooke had already shown that this motion matched that of the shadow of a gnomon over the face of a sundial, but here he also referred to the device as constituting a ‘universal joint’ and described its construction.18

**ANGLED COUPLINGS**

Before proceeding further with applications of this device it is necessary to look back to an earlier phase in the history of mechanisms. The need to transmit a rotary motion from a primary shaft to a second shaft at an angle to the first (rather than directly in line or parallel to it) must have arisen repeatedly since ancient times, and been solved by a number of anonymous artisans. An example cited by the Jesuit father Gaspar Schott was the 1354 clock in Strasbourg Cathedral, where the dial face was situated some way above and to the side of the driving mechanism. Schott explains in a work published in 1664 that an angled drive could have been achieved with bevel gears, but was more simply accomplished with a chain of devices individually known as a *paradoxum*. He illustrates this mechanism with the woodcut reproduced here as figure 7, crediting it to an unpublished manuscript *Chronometria Mechanica Nova* by a deceased anonymous author identified only as ‘Amicus’. Schott cites him as his authority to state specifically that if one shaft moves in a uniform circular motion, the other will also be compelled to rotate uniformly. The *paradoxum* can be seen to be made up of the same two forks linked by a four-armed cross that characterize Hooke’s ‘universal’: it is hard to avoid the conclusion that he saw Schott’s work between 1667 and 1675 and incorporated the improved construction in his second sundial delineator and universal joint. (It is significant that Hooke never claimed to have invented the universal—although he was quite happy to let people think that he had!) However, his experience with the first sundial delineator of 1667 would have alerted him to the error of the ‘Amicus’—Schott claim that it was a ‘constant-velocity’ joint.

![Figure 6. Robert Hooke’s refined instrument for delineating sundials (1676).](image-url)
Hooke’s 1667 instrument was far too light and crude to have functioned satisfactorily as a flexible coupling to transmit significant torque through an angle, although he does say that it could ‘facilitate wheel-work and have other mechanical uses’. However, the metamorphosed form of 1675 eventually proved far more important as a rotating angled coupling than as a sundial delineator. It was Hooke’s study of the motion of the universal joint, and his advocacy and application of the mechanism, that led to its becoming commonly known as a ‘Hooke’s joint’ to millwrights—at least in England. As the joint brisé it is said to have been employed in Dutch windmills for coupling the sails to an Archimedean screw for raising sediment-laden water from drainage ditches. In this application it would not matter (and perhaps was not always realized) that the output velocity was not uniform.

MATHEMATICAL ANALYSIS

Hooke was a competent mathematician, but the spherical trigonometry of his time was insufficiently developed to permit analysis of the kinematics of the universal joint. Not until 1845 did Poncelet prove that

\[ \tan \beta = \tan \alpha \cos \mu, \]

where \( \alpha \) is the angular position of the input shaft, \( \beta \) is the angular position of the output shaft, \( \mu \) is the inclination of one shaft to the other—the ‘angle of articulation’.

---

**Figure 7.** The universal joint (paradoxum) of Schott (1664), after ‘Amicus’. (Reproduced by courtesy of the Library of the Museum of the History of Science, Oxford.)
The derivation is repeated by Willis and—in view of the importance of Hooke’s joints in automotive transmissions—in modern specialist texts devoted to the latter. When applied to sundials, figure 6 shows that setting the angle of inclination to the latitude produces a vertical dial in which

\[
\tan \beta = \tan \alpha \cos \phi = \tan \alpha \sin(90 - \phi).
\]

For a horizontal dial, it may be shown that

\[
\tan \beta = \tan \alpha \cos(90 - \phi) = \tan \alpha \sin \phi.
\]

This expression is identical with the ‘equation of the sundial’ quoted in modern works on their construction. Cundy and Hogg also derive this expression for Hooke’s joint, and use it to calculate the layout of the elliptical (or analemmatic) dial.

**Practical verification**

**Models**

Working models of Hooke’s joints were constructed to illustrate both the ‘cross’ and the ‘disc’ forms of interior member. They are shown in figure 8, along with commercial examples in steel and acetal plastic. The last are free of backlash and considerably less expensive, but as the cross is reduced to a block buried within the forks it is harder to demonstrate its motion.

**Test rig**

Apparatus for the study of couplings at various angles of articulation was built, and is shown in figure 9. It was initially used to find the maximum angle of articulation allowed by the various constructions shown in figure 8. An inclination of 90° would obviously not be possible as a result of geometrical constraint, but smaller angles are limited by mechanical design when the tip of one fork contacts the base of the other near its shaft. Results are shown in Table 1. It would be possible to increase the range of the curved-arm versions to 60° by reducing and curving the tips of the forks, but this figure seems to represent a practical limit for a single Hooke’s joint.

**Phase**

Points at which the output velocity becomes maximal or minimal are 90° apart. Engineers say that this denotes the phase of the Hooke’s joint, and a minimum velocity occurs when the internal cross is normal to the output shaft.

**Amplitude**

The discrepancy from uniform spacing of the hour lines on a sundial decreases as latitude increases. This is reflected by the increasing amplitude of the departure from uniform motion as the articulation angle of a Hooke’s joint is increased from zero (where the two shafts are in line) to 60° (figure 10). It will be seen that the curves are slightly asymmetrical, and peak at angles of 45–55° and subsequent corresponding points. Variation is ±10° at the commercially recommended maximum articulation of 45° for rapidly rotating couplings. This rises to ±20° for shafts inclined at 60° to each other.
Figure 8. Demonstration models of ‘single’ Hooke’s joints. From top to bottom: with a cross-shaped interior member; with a disc-shaped interior member; two commercial versions.

Figure 9. Apparatus for studying the motion of single or double Hooke’s joints at various angles of articulation.
Sundial delineation

The commercial single Hooke’s joint in acetal plastic was fixed in the measuring apparatus of figure 9, the phase and pointers were set to zero, and the output yoke was fixed vertically along the 12–12 line. The angle of articulation was set to a chosen latitude. It was then confirmed that moving the input shaft in equal $15^\circ$ increments caused the pointer on the output shaft to agree with the hour lines on a direct vertical dial previously drawn by conventional methods for the given latitude.

Horizontal dials may be delineated by setting the articulation angle to the co-latitude of the site and arranging the output shaft to be vertical. The yoke on this shaft should be parallel to the 12–12 line.

In both cases the direction of the numeration must be appropriate for the hemisphere: in the Northern Hemisphere the shadow moves anticlockwise over a vertical dial, and clockwise on a horizontal dial.

### Complex Hooke’s joints

**Single, but with adjustable arms**

Hooke’s first proposed application of the basic all-metal universal was to provide a drive in azimuth (from a polar axis) to an astronomical quadrant mounted in such a way that its own construction allowed manual adjustment in altitude (figure 11). The simple universal would not have worked in this situation: to track any object (including the real Sun) over the celestial

<table>
<thead>
<tr>
<th>construction</th>
<th>maximum articulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>reconstruction of Hooke’s original sundial delineator</td>
<td>$35^\circ$</td>
</tr>
<tr>
<td>modern acetal version</td>
<td>$53^\circ$</td>
</tr>
<tr>
<td>curved-arm versions incorporating cross or disc</td>
<td>$57^\circ$</td>
</tr>
<tr>
<td>modern steel units of cylindrical section</td>
<td>$60^\circ$</td>
</tr>
</tbody>
</table>

Figure 10. Variation of angular difference between input and output shafts of a Hooke’s joint for three values of inclination.
Hooke’s joint and sundials

Figure 11. Hooke’s proposed driven altazimuth mounting for a quadrant (1674).

sphere requires that account be taken of its declination. The simple (symmetrical) Hooke’s joint is suitable only for an object on the celestial equator, where its declination is zero. However, it tracks the shadow of a gnomon over a sundial because, as explained above, the polar style results in a shadow direction that is independent of the Sun’s declination. To cope with any object in the real sky, Hooke designed a universal in which the angle of the pivot pin with respect to either shaft was infinitely variable (figure 12) and so could be set manually to the declination of the chosen object. This elegant drawing27 is commonly chosen to illustrate Hooke’s joint—but rarely with any note or explanation of why it is of adjustable asymmetry. Like so many of Hooke’s inventions it is doubtful whether the proposed altazimuth drive was ever built: his driven equatorial drive28 would have been far more valuable to astronomers.

The double Hooke’s joint

Hooke himself realized that there could be circumstances in which the periodic variation in speed characterizing the standard form of coupling would be deleterious. His solution was to employ two universals, one at each end of a common intermediate shaft, to give a ‘double’
joint (figure 13). The units may be assembled ‘in phase’, 90° out of phase, or in any intermediate position. The orientation resulting in a transmitted motion 90° out of phase with the input, combined with parallel input and output shafts (or equal angles of inclination to the intermediate shaft) negates the variation, and the whole assembly is commonly known nowadays as a constant-velocity coupling. They are available commercially in a range of sizes. The addition of a splined assembly in the intermediate shaft allows back and forth motion and has provided a coupling that, centuries after Hooke’s time, has proved enormously successful in the automobile. It allows power to be transmitted from engine to wheels even when their relative positions are changing as a result of going over bumps and hollows in the road. Millions are made every year.

THE SUNDIAL-CLOCK

Hooke himself first suggested, and later discussed quite extensively, driving the input shaft of his sundial delineator with a 24-hour clock. This would permit the relatively easy construction of a nonlinear mechanical ‘clock’ that would track the motion of a gnomon’s shadow over the face of a sundial and take over the time-telling function when clouds covered the Sun or at night. Nowadays, floodlighting would enable the clock to be read at any time, so
the dial should embody the sundial ‘night hours’ drawn by extending the usual hour lines through the centre. Such a mechanism is more complicated than an ordinary clock for the following reasons.

(i) Its direction of rotation must be appropriate for the location and orientation. For a public sundial-clock a low-voltage synchronous electric gearmotor with a friction clutch might be advisable.
(ii) The angle of articulation of the Hooke’s joint must also be correct for the site and orientation. This will, in principle, require the input shaft to be set at the latitude angle along the polar axis, but in cramped surroundings the motor and primary drive shaft may be rotated around the output shaft to best suit the conditions.
(iii) The phase must also be set to suit both dial and orientation of the drive by noting the position of the output yoke. Vertical dials will always have their 12–12 line vertical.
(iv) ‘The time’ must be set after temporarily loosening the screw securing the rear portion of the universal to the 24-hour clock drive. Alternatively, a friction clutch could be incorporated between the universal joint and the motor. It is better to set on a convenient hour than to attempt interpolation in the nonlinear intervals between numerals.

Figure 14 shows the rear elevation of a model sundial-clock for a vertical wall facing south. It is driven via a single Hooke’s joint.

SUNDIAL-CLOCKS FOR DECLINING VERTICAL WALLS

The two shafts meeting at the centre of a single Hooke’s joint must, by geometry, fall in one plane. When the input shaft is aligned with the pole, the output shaft will necessarily be in the same plane and must be arranged horizontally and pointing south to give an articulation equal to the latitude. This will be appropriate to drive a sundial-clock on a vertical wall facing
south, as shown above. What the simple mechanism is unable to do is subsequently move sideways out of that plane to pierce a wall declining east or west, and follow the associated asymmetrical dial.

Hooke proposed\textsuperscript{30} that the remedy was to use his ‘double universal’. An assembly for a vertical wall at latitude $52^\circ$ N declining $20^\circ$ E is shown isolated in figure 15. It uses two separate couplings, but a unit with an integral intermediate shaft would be more compact and could not be misaligned in error. The input shaft must be held at the latitude angle to an intermediate shaft maintained in a horizontal position. This assembly was checked experimentally to match the graduations on a predrawn dial computed\textsuperscript{23} for these conditions. Again, the fork of the final shaft must parallel the vertical 12–12 line on the dial.
It is thought that a public sundial-clock in a suitable location would be an appropriate memorial to Robert Hooke. A possible design is shown in figure 16, and a site is being sought.  

**HOOKE MEMORIAL SUNDIAL-CLOCK**

Figure 15. Arrangement of two Hooke’s joints and three shafts required for a sundial-clock on a declining vertical wall.
Willis also pointed out that—as is commonly found with useful mechanisms—nature achieved it first. The claws of a crab and other crustaceans embody a hinged plate of cartilage that acts as an asymmetric cross in a Hooke’s joint, permitting articulation around two inclined axes (figure 17).

A POSTSCRIPT ON GIMBALS

Gimbals are nowadays generally understood to be a system of three concentric rings pivoted at 90° to each other. This arrangement has the valuable characteristic that an object such as a lamp, mariner’s compass or chronometer mounted at the centre, with its centre of gravity
below the crossing point of the axes, is maintained parallel to its normal position even when
the support is moved into different angular positions. Swivelling gimbals, although known in
antiquity, became more generally familiar in Europe through the writings of the
mathematician Hieronymus Cardano.\textsuperscript{33} In 1557 he showed specifically that the above ‘ring
joints’ gave rise to three degrees of freedom, and the arrangement is therefore sometimes
referred to as a ‘Cardan suspension’. Although it is related to Hooke’s universal joint, and is
occasionally confused with it, their functions are quite different.

ACKNOWLEDGEMENTS

I am grateful for the advice and encouragement of Chris Lusby-Taylor.

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